Lösungen PK 09

B2 (6)

2.1. ges.: A, G (1 LE = 1 m)

Lös.: $\underline{A(-2{,}75 \mid 0 \mid 0)}$ ✓ } aus Grafik abgelesen (2)

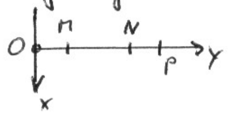
$\underline{G(2{,}75 \mid 8{,}50 \mid 5{,}00)}$ ✓

2.2. geg.: $M(0 \mid 2 \mid 7{,}5)$

ges.: Nachweis für M

Lös.:

1) Gesamthöhe des Hauses 7,5 m $\curvearrowright$ $\underline{z_M = 7{,}5}$ ✓
2) wegen Symm. des Hauses $\curvearrowright$ $\underline{x_M = 0}$
3) wegen Symm. der Firstlinie zur Hausbreite gilt:



$\overline{OP} = \overline{BC} = 8{,}50\,m$

$\overline{BC} - \overline{MN} = 8{,}50\,m - 4{,}50\,m = \underline{4\,m}$

$\curvearrowright \overline{OM} = \overline{NP} = 2\,m$

$\curvearrowright \underline{y_M = 2}$ ✓

ges.: $\sphericalangle(E_\Delta, E_\square)$; $E_\Delta = E(EFM)$; $E_\square = E(EFGH)$

Lös: E_Δ: $E(-2{,}75 \mid 0 \mid 5)$; $F(2{,}75 \mid 0 \mid 5)$; $M(0 \mid 2 \mid 7{,}5)$

$\curvearrowright \quad \vec{x} = \overrightarrow{OE} + r \cdot (\overrightarrow{EF}) + s(\overrightarrow{EM})$

$$\vec{x} = \begin{pmatrix} -2{,}75 \\ 0 \\ 5 \end{pmatrix} + r \underbrace{\begin{pmatrix} 5{,}5 \\ 0 \\ 0 \end{pmatrix}}_{\vec{a}} + s \underbrace{\begin{pmatrix} +2{,}75 \\ 2 \\ 2{,}5 \end{pmatrix}}_{\vec{b}}$$

$$\curvearrowright \quad \vec{a} \times \vec{b} = \vec{n}_{E_\Delta} = \begin{vmatrix} i & 5{,}5 & 2{,}75 \\ j & 0 & 2 \\ k & 0 & 2{,}5 \end{vmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ -13{,}75 \\ 11 \end{pmatrix}}} ✓$$

$E_\square$: oder auch x-y-Ebene: $\underline{\underline{\vec{n}_{E_\square} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}}$

$$\curvearrowright \underbrace{\sphericalangle(E_\Delta, E_\square)}_{\alpha} : \cos \sphericalangle(E_\Delta, E_\square) = \left| \frac{\vec{n}_\Delta \circ \vec{n}_\square}{|\vec{n}_\Delta| \cdot |\vec{n}_\square|} \right|$$

$$\cos \sphericalangle() = \left| \frac{11}{\sqrt{(-13{,}75)^2 + 11^2} \cdot 1} \right| = 0{,}6247$$

$$\curvearrowright \sphericalangle(E_\Delta, E_\square) \approx \underline{51{,}3^\circ} ✓$$

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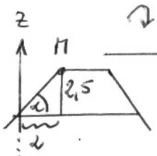
oder:



$\curvearrowright \tan \alpha = \frac{2{,}5}{2} \curvearrowright \alpha = 53{,}3^\circ$

zu 2.2. ges.: Nachweis für J (7)

Lös.: $J \in g(M,F)$

$g(M,F)$: $\vec{x} = \vec{OM} + t \cdot \vec{MF}$

$\vec{x} = \begin{pmatrix} 0 \\ 2 \\ 7{,}5 \end{pmatrix} + t \cdot \begin{pmatrix} 2{,}75 \\ -2 \\ -2{,}5 \end{pmatrix}$ ✓

$J \rightarrow \begin{pmatrix} \frac{15}{4} \\ -\frac{8}{11} \\ \frac{45}{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 7{,}5 \end{pmatrix} + t \begin{pmatrix} 2{,}75 \\ -2 \\ -2{,}5 \end{pmatrix}$ (6)

$\frac{15}{4} = 2{,}75t \quad \wedge \quad t = \frac{15}{11}$]

$-\frac{8}{11} = 2 - 2t \qquad \underline{\underline{-\frac{8}{11}}} = 2 - 2 \cdot \frac{15}{11} = \underline{\underline{-\frac{8}{11}}}$

$\frac{45}{11} = 7{,}5 - 2{,}5t \qquad \underline{\underline{\frac{45}{11}}} = 7{,}5 - 2{,}5 \cdot \frac{15}{11} = \underline{\underline{\frac{45}{11}}}$ ✓

wahr $\wedge$ $\underline{J \in g(MF)}$

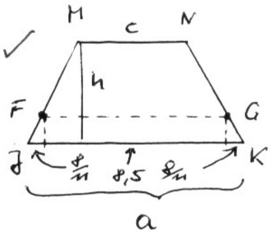
2.3. ges.: A_{JKNM} = Trapez A_T

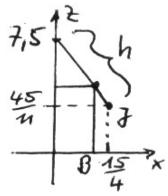
Lös.: $a = 2 \cdot \frac{8}{11} + 8{,}5 = \underline{\underline{9{,}955\,m}}$ ✓

$c = \overline{MN} = \underline{\underline{4{,}5\,m}}$

$h = \sqrt{\left(7{,}5 - \frac{45}{11}\right)^2 + \left(\frac{15}{4}\right)^2}$

$\underline{\underline{h \approx 5{,}07\,m}}$ ✓

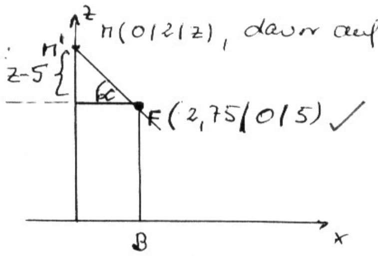




(3)

$\wedge \quad A_T = \frac{a+c}{2} \cdot h = \frac{9{,}955 + 4{,}5}{2} \cdot 5{,}07\,m^2 \approx \underline{\underline{36{,}63\,m^2}}$ ✓

2.4. geg.:



$M(0|2|z)$, davon auf der z-Achse $M'(0|0|z)$

(3)

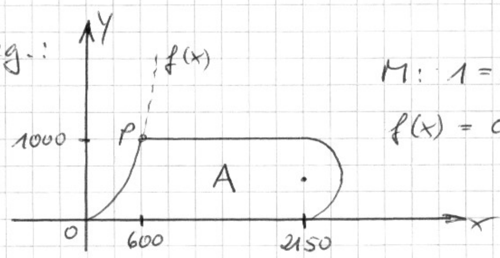
ges.: z in Abhängigkeit von α

Lös.: $\tan\alpha = \frac{GK}{AK} = \frac{z-5}{2{,}75}$ ✓

$\wedge \quad z = 2{,}75 \tan\alpha + 5 \quad \wedge \quad \underline{\underline{M(0|2|2{,}75\tan\alpha + 5)}}$ ✓

D2) (14)

a) geg.:



M: 1 = 1mm

$f(x) = ax^2$

ges.: A

Lös.: $A = \int_0^{600} f(x)\,dx + 1550 \cdot 1000 + \frac{1}{2}\pi \cdot 500^2$ ✓

NR: $f(x)$ mit $P(600|1000)$

$\Rightarrow 1000 = a \cdot 600^2 \Rightarrow a \approx 0{,}0028 \Rightarrow \underline{f(x) = 0{,}0028x^2}$ ✓

$A = \int_0^{600} 0{,}0028x^2\,dx + 1550000 + \frac{1}{2}\pi \cdot 500^2$ ✓ (5)

$A \approx 2144299{,}1\ mm^2 \approx \underline{\underline{2{,}14\ m^2}}$ ✓

b) geg.: Abbaubereich ≙ Ebene $E^*(A, B, T_\alpha, S_\alpha) = E_\alpha$

mit $A(600|500|0)$; $B(200|500|0)$ (1 = 1m)

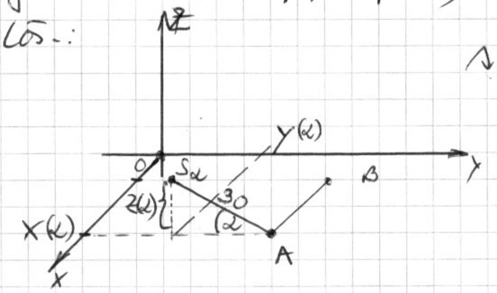
Ausleger $\Rightarrow \overline{AS_\alpha} \parallel yz$-Ebene; $|\overrightarrow{AS_\alpha}| = 30m$

$0° \leq \alpha \leq 50°$

bei $\alpha = 0°$ ist $E_{0°} \equiv xy$-Ebene

ges.: $S_\alpha(x(\alpha)|y(\alpha)|z(\alpha))$

Lös.:



$\Rightarrow \sin\alpha = \frac{z(\alpha)}{30}$

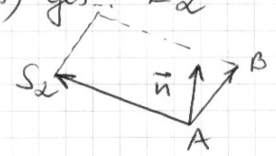
$\Rightarrow \underline{z(\alpha) = 30 \cdot \sin\alpha}$ ✓

$\cos\alpha = \frac{500 - y(\alpha)}{30}$

$\Rightarrow \underline{y(\alpha) = 500 - 30\cos\alpha}$

$\Rightarrow \underline{\underline{S_\alpha(600|500 - 30\cos\alpha|30\sin\alpha)}}$ ✓

zu b) ges.: E_α (15)



$$\vec{n} = \vec{AB} \times \vec{AS_\alpha} = \begin{vmatrix} \vec{i} & -400 & 0 \\ \vec{j} & 0 & -30\cos\alpha \\ \vec{k} & 0 & 30\sin\alpha \end{vmatrix}$$

$$\curvearrowright \vec{n} = \begin{pmatrix} 0 \\ -(-400 \cdot 30\sin\alpha) \\ 400 \cdot 30\cos\alpha \end{pmatrix} = \begin{pmatrix} 0 \\ 12000\sin\alpha \\ 12000\cos\alpha \end{pmatrix}$$

$$\curvearrowright E_\alpha: \quad 0x + 12000\sin\alpha\, y + 12000\cos\alpha\, z = d$$

$$A \rightarrow \quad 0 \cdot 600 + 12000\sin\alpha \cdot 500 + 12000\cos\alpha \cdot 0 = d$$

$$\curvearrowright d = 6000000\sin\alpha$$

$$\curvearrowright E_\alpha: \quad 12000\sin\alpha\, y + 12000\cos\alpha\, z = 6000000\sin\alpha$$

$$\underline{\underline{12\sin\alpha\, y + 12\cos\alpha\, z = 6000\sin\alpha}} \quad \text{bzw. } (:12)$$

$$\underline{\underline{\sin\alpha\, y + \cos\alpha\, z = 500\sin\alpha}} \checkmark$$

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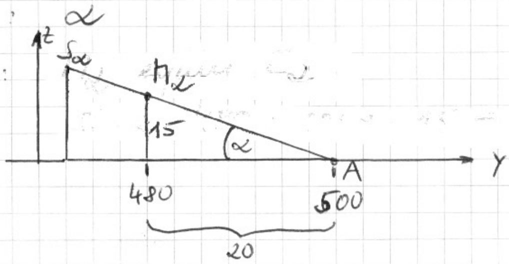
geg.: $M_\alpha(350 / 480 / 15)$

ges.: α

Lös.:

$\curvearrowright$



✓

(5)

$$\curvearrowright \tan\alpha = \frac{15}{20} \quad \curvearrowright \underline{\underline{\alpha \approx 37^\circ}} \checkmark$$

ODER: M_α in E_α einsetzen:

$$\sin\alpha \cdot 480 + \cos\alpha \cdot 15 = 500 \cdot \sin\alpha$$

CP: $\alpha = \pi k + 0{,}6435$

für $k = 0$ folgt $\underline{\underline{\alpha = 0{,}6435 \mathrel{\hat=} 37^\circ}}$